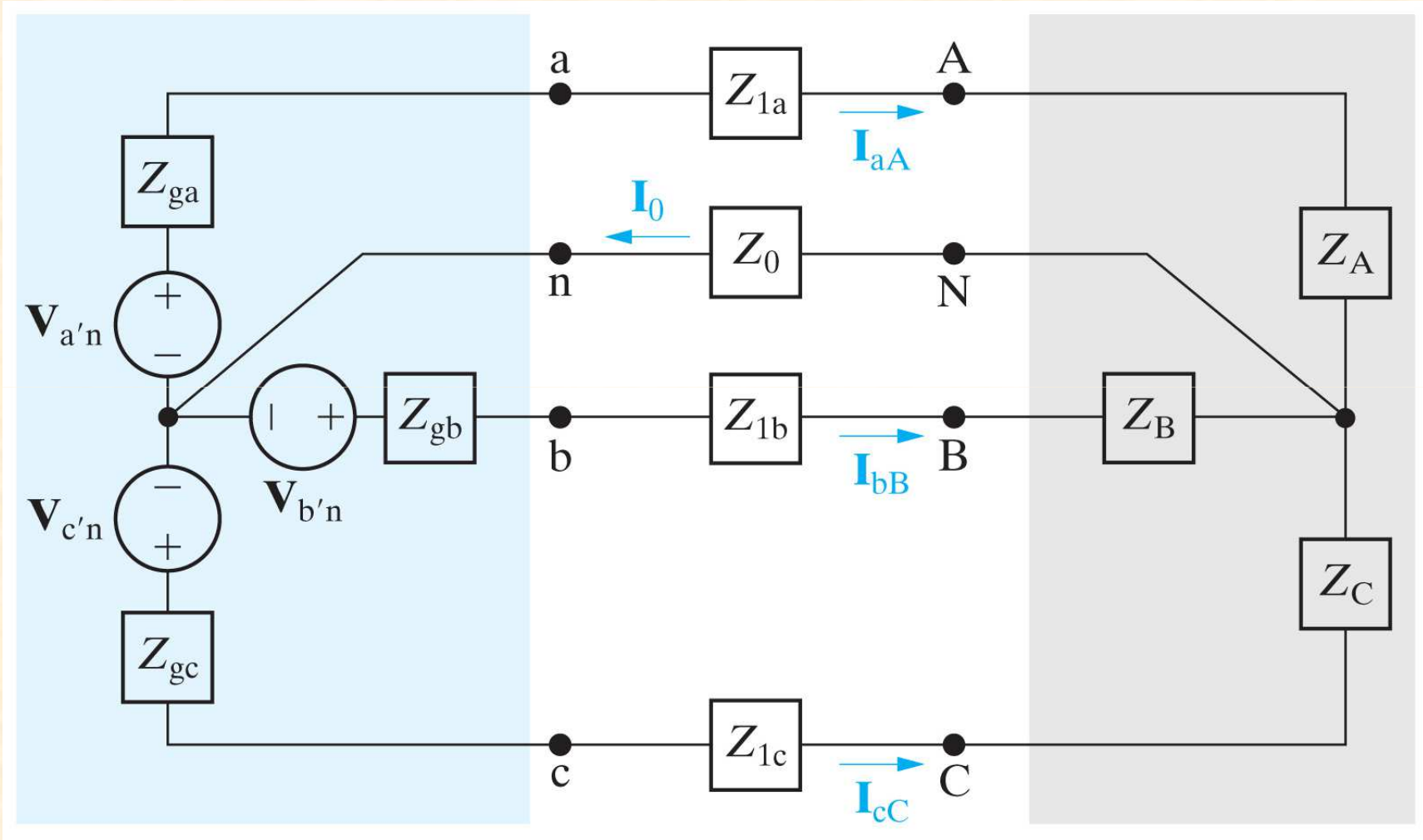
# Y-Y Three-Phase System (Four Wire)



# Y-Y Three-Phase System (Four Wire)

- $Z_g$ represents the internal generator impedance per phase
- $Z_l$ represents the impedance of the line connecting the generator to the load
- $Z_{A,B,C}$ represents the load impedance per phase
- $Z_o$ represents the impedance of the neutral conductor

# Neutral Voltage for Y-Y System

➢Using source neutral as a reference, the Node-Voltage equation at node N can be written as:

$$\frac{\mathbf{V}_N}{Z_o} + \frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_A + Z_{la} + Z_{ga}} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_B + Z_{lb} + Z_{gb}} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_C + Z_{lc} + Z_{gc}} = 0.$$

➢ ***For a balance three-phase system;***

✓ Three-phase voltages are balanced,

✓ $Z_{ga} = Z_{ga} = Z_{ga}$ , $Z_{la} = Z_{la} = Z_{la}$ **and** $Z_A = Z_B = Z_C$

$$Z_\phi = Z_A + Z_{la} + Z_{ga}$$

# Neutral Voltage for Y-Y System

➢ **The neutral voltage can be given by:**

$$\mathbf{V}_N\left(\frac{1}{Z_o}+\frac{3}{Z_\phi}\right)=\frac{\mathbf{V}_{a'n}+\mathbf{V}_{b'n}+\mathbf{V}_{c'n}}{Z_\phi}.$$

❑ As the three-phase voltages are balanced (i.e. $\mathbf{V_{an}}+\mathbf{V_{bn}}+\mathbf{V_{cn}}=\mathbf{0}$), therefore the neutral voltage must be equals zero

$$\mathbf{V}_N=0.$$

# Neutral Current for Y-Y System

- The three-phase line currents can be written as:

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_A + Z_{la} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_\phi};$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_B + Z_{lb} + Z_{gb}} = \frac{\mathbf{V}_{b'n}}{Z_\phi};$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_c + Z_{lc} + Z_{gc}} = \frac{\mathbf{V}_{c'n}}{Z_\phi}.$$

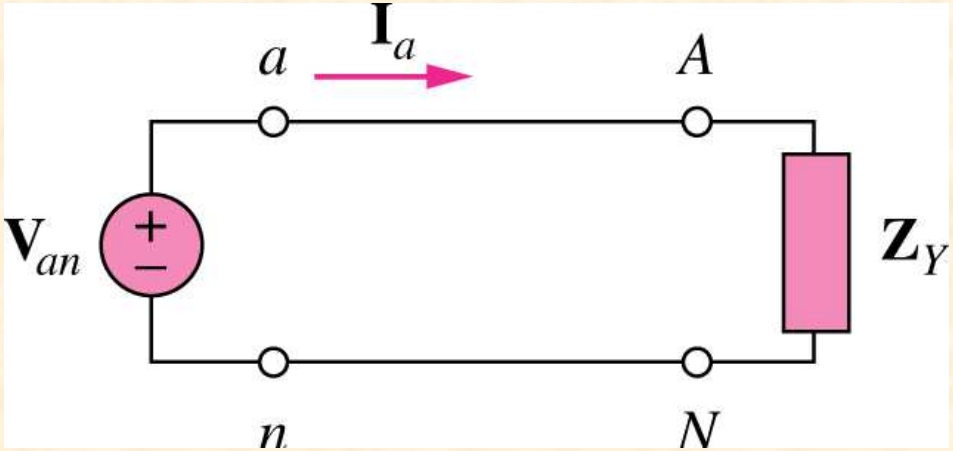
- **The neutral current can be given by:**

$$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC},$$

- As the three-phase line currents are balanced (i.e. $\mathbf{I_{ab}} + \mathbf{I_{bc}} + \mathbf{I_{ca}} = 0$), therefore the neutral current must be equals zero $(I_0 = 0)$

# Single Phase Equivalent of Balanced Y-Y Connection

- Balanced three phase circuits can be analyzed on "per phase " basis.
- Look at one phase, say phase *a* and analyze the single phase equivalent circuit
- Because the circuit is balanced, the other phase values can be easily obtained using their phase relationships




$$I_a = \frac{V_{an}}{Z_Y}$$

## ➢Example on Y-Y System

A three-phase, positive-sequence, Y-connected generator has an impedance of $0.2 + j0.5\ \Omega/\phi$. The internal phase voltage of the generator is 120 V. The generator feeds a balanced, three-phase, Y-connected load having an impedance of $39 + j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the three line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

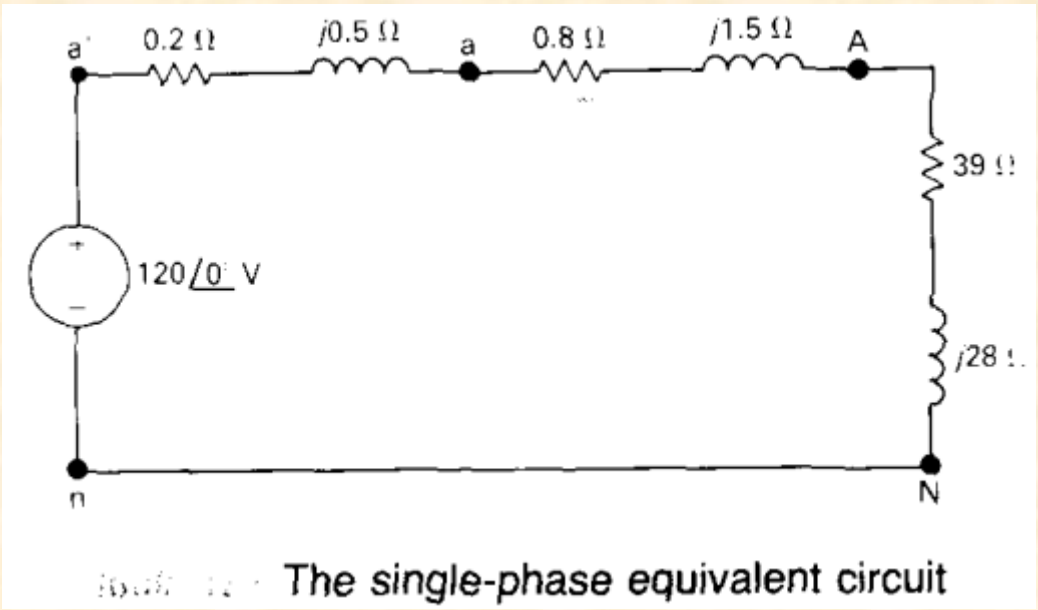
c) Calculate the three line-to-neutral voltages at the load $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, $\mathbf{V}_{CN}$.

d) Calculate the line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ at the terminals of the load.

e) Calculate the line-to-neutral voltages at the terminals of the generator $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$.

f) Calculate the line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ at the terminals of the generator.

g) Repeat (a)–(f) for a negative phase sequence.




The single-phase equivalent circuit

b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120/0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120/0^\circ}{40 + j30} = 2.4/-36.87^\circ \text{ A}.$$

For a positive phase sequence,

$$\mathbf{I}_{bB} = 2.4/-156.87^\circ \text{ A};$$

$$\mathbf{I}_{cC} = 2.4/83.13^\circ \text{ A}.$$

c) The line-to-neutral voltage at the A terminal of the load is

$$\mathbf{V}_{AN} = (39 + j28)(2.4/-36.87^\circ)$$

$$= 115.22/-1.19^\circ \text{ V}.$$

For a positive phase sequence,

$$\mathbf{V}_{BN} = 115.22/-121.19^\circ \text{ V};$$

$$\mathbf{V}_{CN} = 115.22/+118.81^\circ \text{ V}.$$

d) For a positive phase sequence, the line-to-line voltages lead the line-to-neutral voltages by 30°; thus

$$\mathbf{V}_{AB} = (\sqrt{3}/30°)\,\mathbf{V}_{AN}$$

$$= 199.58/28.81°\ \text{V};$$

$$\mathbf{V}_{BC} = 199.58/-91.19°\ \text{V};$$

$$\mathbf{V}_{CA} = 199.58/148.81°\ \text{V}.$$

e) The line-to-neutral voltage at the a terminal of the source is

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4/-36.87°)$$

$$= 120 - 1.29/31.33°$$

$$= 118.90 - j0.67$$

$$= 118.90/-0.32°\ V.$$

For a positive phase sequence,

$$\mathbf{V}_{bn} = 118.90/-120.32°;$$

$$\mathbf{V}_{cn} = 118.90/119.68°\ \text{V}.$$

f) The line-to-line voltages at the source terminals are

$$\mathbf{V}_{ab} = (\sqrt{3}/30^\circ)\,\mathbf{V}_{an}$$
$$= 205.94/29.68^\circ \text{ V};$$
$$\mathbf{V}_{bc} = 205.94/-90.32^\circ \text{ V};$$
$$\mathbf{V}_{ca} = 205.94/149.68^\circ \text{ V}.$$

g) Changing the phase sequence has no effect on the single-phase equivalent circuit. The three line currents are

$$\mathbf{I}_{aA} = 2.4/-36.87^\circ \text{ A};$$
$$\mathbf{I}_{bB} = 2.4/83.13^\circ \text{ A};$$
$$\mathbf{I}_{cC} = 2.4/-156.87^\circ \text{ A}.$$

The line-to-neutral voltages at the load are

$$\mathbf{V}_{AN} = 115.22\angle -1.19^\circ \text{ V};$$

$$\mathbf{V}_{BN} = 115.22\angle +118.81^\circ \text{ V};$$

$$\mathbf{V}_{CN} = 115.22\angle -121.19^\circ \text{ V}.$$

For a negative phase sequence, the line-to-line voltages lag the line-to-neutral voltages by 30°:

$$\mathbf{V}_{AB} = (\sqrt{3}\angle -30^\circ \mathbf{V}_{AN}$$

$$= 199.58\angle -31.19^\circ \text{ V};$$

$$\mathbf{V}_{BC} = 199.58\angle 88.81^\circ \text{ V};$$

$$\mathbf{V}_{CA} = 199.58\angle -151.19^\circ \text{ V}.$$

The line-to-neutral voltages at the terminals of the generator are

$$\mathbf{V}_{an} = 118.90\angle -0.32^\circ \text{ V};$$

$$\mathbf{V}_{bn} = 118.90\angle 119.68^\circ \text{ V};$$

$$\mathbf{V}_{cn} = 118.90\angle -120.32^\circ \text{ V}.$$

The line-to-line voltages at the terminals of the generator are

$$\mathbf{V}_{ab} = (\sqrt{3}\angle -30^\circ)\mathbf{V}_{an}$$

$$= 205.94\angle -30.32^\circ \text{ V};$$

$$\mathbf{V}_{bc} = 205.94\angle 89.68^\circ \text{ V};$$

$$\mathbf{V}_{ca} = 205.94\angle -150.32^\circ \text{ V}.$$